The effect of cost sharing on input choice in sharecropping contracts: Theory

(this version March 2006)

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1 Why does cost sharing exist: A review of the literature

It is widely observed that besides the output in a share contract, the costs of at least one of the inputs, typically fertilizer, are shared between the landlord and the tenant. One question that is crucial for the analysis of the efficiency of sharecropping contracts is whether there is an effect of cost-sharing on the intensity with which the inputs in question are applied to the crop, and how this effect works. A brief account of the relevant theory will help to clarify this point.[1]

It was long believed that, under certainty, a cost-sharing arrangement by which input costs are shared between the landlord and the tenant at the same rate as output would result in the optimum resource allocation, since the share tenant's disincentive to apply inputs under output sharing would be exactly offset by the subsidy to the input costs under the equal output and cost-sharing rule. But Braverman and Stiglitz (1986) make the point that if cost sharing is feasible, then it must be possible for the landlord to observe the level of inputs, and, if the level of inputs is observable, it is at least feasible that the contract specify the level of inputs. Given this, they show that with a linear sharecropping contract (comprising an output share, a cost share, and a fixed payment), there is no reason for the landlord to opt for cost-sharing, since he does no worse and no better by specifying the input level than he does by using a cost-sharing contract, even if the tenant is risk averse. So why, then, do cost-sharing arrangements exist? Braverman and Stiglitz come up with a resolution to this paradox: asymmetric information between the landlord and the tenant. Since the optimal level of input changes in response to variations in weather and geographically, according to the nature of the

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[1] Surveys of the theoretical literature on cost-sharing are provided by Singh (1989) and Hayami and Otsuka (1993).

soil and other local conditions, the tenant will often be in a better position to make decisions concerning the level of inputs. If these changes and differences in circumstances are observable to the tenant but not to the landlord, then a contract which induces the tenant to adjust the input to the altered circumstances will be preferred by the landlord. In this sense, a cost-sharing contract is a more flexible contract than a fixed-quantity contract. A further explanation for the existence of cost-sharing arrangements is provided by Bardhan and Singh (1987), using a related argument. They show that if the landlord cannot monitor the tenant's actions, and if the tenant is able to resell the input which is cost-shared with the landlord at a price lower than the landlord's opportunity costs for this input, then there is indirect cost sharing at the margin. Relating these results to the task of empirically settling the question of whether the Marshallian or the monitoring approach is the correct one, one can conclude that if the above theories are valid under the actually prevailing conditions, then the existence of cost-sharing in the data is an indicator that there are monitoring problems.[2]

2 The model

Knowing now why there is cost-sharing, the next question is, how exactly does the presence of cost-sharing influence the tenant's decisions concerning input use. Using the Braverman-Stiglitz framework, I assume that there is asymmetric information between the landlord and the tenant concerning some aspect of the technology which varies with the state of nature and which affects the productivity of an input such as fertilizer. Consider a risk averse tenant with a twice differentiable concave utility function defined on income, $u(y)$.

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[2] Hayami and Otsuka propose another explanation for cost sharing which has not yet been formalized and which makes use of the argument that the inputs supplied under cost sharing can be regarded as de facto production loans.

The tenant is assumed to control an input *x*, the amount of which can be observed by the landlord; but the costs of writing a contract specifying the level of input corresponding to each state of nature are assumed to be prohibitive. Braverman-Stiglitz show that under these assumptions, in general, the optimal contract will entail some degree of cost sharing. Since Braverman-Stiglitz do not investigate in detail how a change in the cost sharing arrangement influences the tenant's input decision, I proceed by modelling the tenant's input choice.

The tenant maximizes his expected utility under the optimal contract by choosing the optimal level of input x^o given his output share, α, his cost share, β, and the price per unit of input, *c*. The level of output is made uncertain by introducing the non-negative, multiplicative scalar θ, which is distributed according to $h(\theta)$ with mean $E[\theta]=1$. Hence, the tenant's maximization problem is:

$$\max_x E\{u(\alpha\theta f(x)-\beta cx)\} \tag{1}$$

where the output price is normalized to unity and the production function is assumed to be strictly concave in *x*. For the production function, I assume the lower and upper inada conditions to hold. The necessary and sufficient condition for an optimal *x* is then:

$$\alpha f'E[u'\theta]-\beta cE[u']=0 \tag{2}$$

Dividing (2) by α and defining $\delta \equiv \frac{\beta}{\alpha}$, I can rewrite (2) as

$$f'E[u'\theta]-\delta cE[u']=0 \tag{3}$$

For an optimally chosen *x* I can write the value function dependent on δ as:

$$V(\delta)=E\{u(\alpha(\delta)\theta f(x^o)-\alpha(\delta)\delta cx^o)\} \tag{4}$$

For the tenant's expected utility at the optimum not to change with a change in δ I must have

$$\frac{\partial V}{\partial \delta}=E\{u'(\alpha'(\delta)\theta f(x^o)-\alpha'(\delta)\delta cx^o-\alpha(\delta)cx^o)\}=0 \tag{5}$$

from which it follows that

$$\alpha'(\delta) = \frac{\alpha}{\frac{E[u'\theta]}{E[u']}\frac{f(x^o)}{cx^o} - \delta} > 0, \qquad (6)$$

where $\frac{E[u'\theta]}{E[u']} < 1$ by virtue of the fact that *u* is strictly concave. The denominator of the right-hand side of (6) is positive, because one knows from (3) that $\delta = \frac{E[u'\theta]}{E[u']}\frac{f'}{c}$ and because for a concave production function one always has $f' < \frac{f}{x}$.

Differentiating the first-order condition in (3) with respect to δ I obtain the comparative static expression which denotes the change of *x* in δ:

$$\frac{\partial x^o}{\partial \delta} = \{cE[u'] + (\alpha'(\delta)\delta + \alpha(\delta))cxE[u''(\theta f' - \delta c)] - \alpha'(\delta)fE[u''\theta(\theta f' - \delta c)]\} / \Delta \qquad (7)$$

where $\Delta = f''E[u'\theta] + \alpha(\delta)E\left[u''(\theta f' - \delta c)^2\right] < 0$. The first term in the numerator of the right-hand side of (7) is the direct effect of a change in δ on x, which is positive, because an increase in δ raises the marginal cost of the input. The other two terms represent the indirect effects which arise because the tenant is risk averse (if the tenant is risk neutral, the last two terms are equal to zero). The second term in (7) is the effect which arises because the change in the costshare changes the tenant's income. For CARA this term is zero, since in this case the tenant's risk aversion does not change with changing income. The third term in (7) is the effect which arises because, for example, the outputshare has to rise to compensate the tenant for a higher costshare, and therefore the risk averse tenant receives a bigger share of the risky output. If then the tenant is sufficiently risk averse, he will employ less of the input to make his stakes at risk smaller. I can state the following proposition:

<u>Proposition1.</u> *If the tenant's utility function satisfies CARA or CRRA then the optimal amount of input chosen by the tenant decreases with increasing* δ*:*

$$\frac{\partial x^o}{\partial \delta} = \{cE[u'] + (\alpha'(\delta)\delta + \alpha(\delta))cxE[u''(\theta f' - \delta c)] - \alpha'(\delta)fE[u''\theta(\theta f' - \delta c)]\} / \Delta < 0 .$$

(Proof see appendix)

Since on a real world farm not only one input will be used in production, but several inputs are used simultaneously in the production of a crop, the question arises whether a change in the costshare-outputshare relation for one input (say fertilizer) has an effect on the amount of another input (say labour) applied by the tenant to his sharecropped plots. Therefore I extend in the following our model to the case of two different inputs, x_1 and x_2. It is assumed that the tenant's costshares for the two inputs are β_1 and β_2, respectively. Further I assume that the production function, $f(x_1, x_2)$, is strictly concave and that the lower and upper inada-conditions hold for both inputs. Then the tenant's maximization problem is:

$$\max_x E\{u(\alpha\theta f(x_1, x_2) - \beta_1 c_1 x_1 - \beta_2 c_2 x_2)\}.$$

Again define $\delta_1 \equiv \frac{\beta_1}{\alpha}$ and $\delta_2 \equiv \frac{\beta_2}{\alpha}$, such that the necessary and sufficient conditions can be written as

$$f_1 E[u'\theta] - \delta_1 c_1 E[u'] = 0, \tag{8a}$$

$$f_2 E[u'\theta] - \delta_2 c_2 E[u'] = 0. \tag{8b}$$

For optimally chosen x_1 and x_2 the value function dependent on δ_1 and δ_2 is

$$V(\delta_1, \delta_2) = E\{u(\alpha(\delta_1, \delta_2)\theta f(x_1^o, x_2^o) - \alpha(\delta_1, \delta_2)\delta_1 c_1 x_1^o - \alpha(\delta_1, \delta_2)\delta_2 c_2 x_2^o)\}. \tag{9}$$

To save space, I will derive the following results only for the costshare-outputshare relation of input x_1, but the results can be easily extended to the costshare-outputshare relation of input x_2. Differentiating (9) with respect to δ_1, setting the result equal to zero, and using (8a) and (8b), I can derive

$$\frac{\partial \alpha}{\partial \delta_1} = \frac{\alpha(\delta_1, \delta_2)}{\frac{1}{c_1}\frac{E[u'\theta]}{E[u']}\left(\frac{f(x_1^o, x_2^o)}{x_1^o} - f_1(x_1^o, x_2^o) - f_2(x_1^o, x_2^o)\frac{x_2^o}{x_1^o}\right)}. \tag{10}$$

The sign of (10) hinges on the sign of the term in parentheses in the denominator. Employing Euler's Formula I can state that

$$\frac{f(x_1^o, x_2^o)}{x_1^o} - f_1(x_1^o, x_2^o) - f_2(x_1^o, x_2^o)\frac{x_2^o}{x_1^o} \gtreqless 0 \quad \Leftrightarrow \quad \begin{cases} DRS \\ CRS \\ IRS \end{cases}.$$

Therefore $\frac{\partial \alpha}{\partial \delta_1} \gtrless 0 \Leftrightarrow \begin{cases} DRS \\ IRS \end{cases}$. For constant returns to scale, (10) is not defined.

Totally differentiating (8a) and (8b) with respect to δ_1 I obtain

$$\frac{\partial x_1^o}{\partial \delta_1} = \frac{c_1 E[u']\{f_{22}E[u'\theta] + \alpha E[u''(\theta f_2 - \delta_2 c_2)^2]\} - D_1 B_2 + D_2 B_1}{\Delta} \tag{11a}$$

and

$$\frac{\partial x_2^o}{\partial \delta_1} = \frac{-c_1 E[u']\{f_{21}E[u'\theta] + \alpha E[u''(\theta f_1 - \delta_1 c_1)(\theta f_2 - \delta_2 c_2)]\} - D_2 A_1 + D_1 A_2}{\Delta}, \tag{11b}$$

with $\Delta = A_1 B_2 - A_2 B_1 > 0$ since the maximization problem of the tenant is strictly concave. (For $A_1, A_2, B_1, B_2, D_1, D_2$ see appendix). The first term in the numerator of (11a) is negative since $B_2 < 0$. If the other terms in the numerator, the indirect effects, are small in magnitude or cancel out, then again the amount of the input chosen in the optimum by the tenant will decrease with an increase in its own costshare-outputshare relation, that is $\frac{\partial x_1^o}{\partial \delta_1} < 0$. For equation (11b) the sign is not clear, even if I assume that the indirect effects are small in magnitude or cancel out. The first term in the numerator of (11b) consists of two parts, the first of which is negative if the cross product of the two inputs is positive, whereas the second part is positive, since $\theta f_1 - \delta_1 c_1$ and $\theta f_2 - \delta_2 c_2$ will have the same sign in each state of nature.

To see this, consider (8a) and (8b), from which it follows that $\frac{\delta_1 c_1}{f_1} = \frac{\delta_2 c_2}{f_2} = \frac{E[u'\theta]}{E[u']} < 1$. Thus, an increase in the costshare-outputshare relation of another input has a negative effect on the amount of the input in question because of the positive cross product, but it has also a positive effect, since the tenant will substitute this input for the input which becomes relatively more costly and the use of which becomes relatively riskier. Therefore, the sign of (11b) will depend on whether the cross product effect or the substitution effect is stronger.

3 Concluding Remarks

These findings represent testable hypotheses. If the results are true, then a tenant whose cost share raises relative to his output share will use a lower amount of the respective input on his sharecropped plot, whereas he will not change his input on his owned plots, so the difference of average input intensities between owned and sharecropped plots will become larger. If there is a monitoring problem, then a raising cost share relative to the output share will have a positive effect on the difference of average input intensities. This argumentation assumes that the tenant cannot divert resources from his tenancy to his owned plots and the other way round, i.e. the production on his leased-in plots is strictly separated from the production on his owned plots. But this may not be the case on real world farms. Bell et al. (1995) identify what they call the 'dilution effect', an effect which arises because some resources are imperfectly tradable, and the tenant therefore must divert such resources, in part, from his own holdings to his tenancy. This effect also extends to tradable resources which are (net) Hicksian complements with non-tradable resources. In this model the presence of this effect would mean that the tenant diverts the non-tradable resource the costshare-outputshare relation of which rises from his tenancy back to his own land, since it becomes relatively more costly to use the respective input on the tenancy. This in turn would imply that in this case the difference between average input intensities on owned and sharecropped plots would even become larger than in the case were no dilution effect is at work.

Appendix

Proof of proposition 1:

This proof borrows from Meyer (1987, pp.428-429).

The sign of the numerator of (7) depends on the signs of $E\left[u''\left(\theta f' - \delta c\right)\right]$ and $E\left[u''\theta\left(\theta f' - \delta c\right)\right]$.

Taking into account that $\frac{\delta c}{f'} = \frac{E[u'\theta]}{E[u']}$ because of (3), I can write:

$E\left[u''\left(\theta f' - \delta c\right)\right] \geq 0 \quad \Leftrightarrow \quad E[u''\theta]E[u'] - E[u'\theta]E[u''] \geq 0 \quad \Leftrightarrow$ $-\int u'dH \int u''\theta dH + \int u'\theta dH \int u''dH \leq 0$, where $H(\theta)$ is the cdf, θ being distributed on the interval $\left[0, \bar{\theta}\right]$. Let θ^* satisfy $\theta^* \int u'dH = \int u'\theta dH$. Thus $\int\left(\theta - \theta^*\right)u'dH = 0$ and the integrand changes sign once from negative to positive. Define $r_A\left(Y(\theta)\right) = -\frac{u''\left(Y(\theta)\right)}{u'\left(Y(\theta)\right)}$ as the coefficient of absolute risk aversion, where $Y = \alpha\theta f(x) - \beta cx$ is the farmer's net income. Using the definition of θ^*, one can now rewrite the expression to be signed as $\int u'dH \int\left(\theta - \theta^*\right)r_A u'dH \leq 0$. Since $\frac{dY}{d\theta} > 0$, I have $\int u'dH \int\left(\theta - \theta^*\right)r_A u'dH \leq 0 \Leftrightarrow \frac{dr_A}{d\theta} \leq 0$.

There from it follows that $\int u'dH \int\left(\theta - \theta^*\right)r_A u'dH \leq 0 \quad \Leftrightarrow \quad \frac{dr_A}{dY} \leq 0$. That is, $\int u'dH \int\left(\theta - \theta^*\right)r_A u'dH \leq 0 \Leftrightarrow \begin{cases} DARA \\ CARA \end{cases}$ or $E\left[u''\left(\theta f' - \delta c\right)\right] \geq 0 \Leftrightarrow \begin{cases} DARA \\ CARA \end{cases}$.

Again taking into account that $\frac{E[u'\theta]}{E[u']} = \frac{\delta c}{f'}$, I can write:

$E\left[u''\theta\left(\theta f'-\delta c\right)\right]\geq 0 \quad \Leftrightarrow \quad E\left[u''\theta^2\right]E\left[u'\right]-E\left[u''\theta\right]E\left[u'\theta\right]\geq 0 \quad \Leftrightarrow$ $\int u''\theta^2 dH\int u'dH-\int u''\theta dH\int u'\theta dH\geq 0$. Let again θ^* satisfy $\theta^*\int u'dH=\int u'\theta dH$. Thus $\int\left(\theta-\theta^*\right)u'dH=0$ and the integrand changes sign once from negative to positive. Define $r_{R\theta}\left(Y(\theta)\right)=-\frac{u''\left(Y(\theta)\right)}{u'\left(Y(\theta)\right)}\theta$. Using the definition of θ^*, I can now rewrite the expression to be signed as $E\left[u''\theta\left(\theta f'-\delta c\right)\right]\geq 0 \Leftrightarrow \int u'dH\int u'r_{R\theta}\left(\theta-\theta^*\right)dH\leq 0$. Since $\frac{dY}{d\theta}>0$, I have $\int u'dH\int u'r_{R\theta}\left(\theta-\theta^*\right)dH\leq 0 \Leftrightarrow \frac{dr_{R\theta}}{d\theta}\leq 0$. Let $r_R=-\frac{u''(Y)}{u'(Y)}Y$ be the coefficient of relative risk aversion. Then $\frac{dr_R}{dY}\leq 0 \Rightarrow \frac{dr_{R\theta}}{d\theta}\leq 0$ and therefore $\int u'dH\int u'r_{R\theta}\left(\theta-\theta^*\right)dH\leq 0 \Leftrightarrow$ $\begin{cases}DRRA\\CRRA\end{cases}$ or $E\left[u''\theta\left(\theta f'-\delta c\right)\right]\geq 0 \Leftrightarrow \begin{cases}DRRA\\CRRA\end{cases}$.

Now, for CARA $E\left[u''\left(\theta f'-\delta c\right)\right]=0$ and $E\left[u''\theta\left(\theta f'-\delta c\right)\right]<0$ because CARA implies IRRA. This makes the numerator of (7) positive and therefore $\frac{\partial x^o}{\partial\delta}<0$. If one assumes that CRRA holds, which implies DARA, then $E\left[u''\left(\theta f'-\delta c\right)\right]>0$ and $E\left[u''\theta\left(\theta f'-\delta c\right)\right]=0$. In this case, too, the numerator of (7) is positive and therefore $\frac{\partial x^o}{\partial\delta}<0$. This proves proposition 1.

Explanations to equations (11a) and (11b):

$A_1=f_{11}E[u'\theta]+\alpha E\left[u''(\theta f_1-\delta_1c_1)^2\right]<0$,

$A_2=f_{21}E[u'\theta]+\alpha E\left[u''(\theta f_1-\delta_1c_1)(\theta f_2-\delta_2c_2)\right]$,

$B_1=f_{12}E[u'\theta]+\alpha E\left[u''(\theta f_1-\delta_1c_1)(\theta f_2-\delta_2c_2)\right]$,

$B_2=f_{22}E[u'\theta]+\alpha E\left[u''(\theta f_2-\delta_2c_2)^2\right]<0$,

$$D_1 = -\left\{ \frac{\partial \alpha}{\partial \delta_1} \delta_1 c_1 x_1 + \frac{\partial \alpha}{\partial \delta_1} \delta_2 c_2 x_2 + \alpha c_1 x_1 \right\} E\left[u''(\theta f_1 - \delta_1 c_1) \right] + \frac{\partial \alpha}{\partial \delta_1} f E\left[u'' \theta (\theta f_1 - \delta_1 c_1) \right],$$

$$D_2 = -\left\{ \frac{\partial \alpha}{\partial \delta_1} \delta_1 c_1 x_1 + \frac{\partial \alpha}{\partial \delta_1} \delta_2 c_2 x_2 + \alpha c_1 x_1 \right\} E\left[u''(\theta f_2 - \delta_2 c_2) \right] + \frac{\partial \alpha}{\partial \delta_1} f E\left[u'' \theta (\theta f_2 - \delta_2 c_2) \right].$$

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